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HEAT TRANSFER IN THIN POROUS FIBROUS MATERIAL: MATHEMATICAL MODELLING AND EXPERIMENTAL VALIDATION USING ACTIVE THERMOGRAPHY

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Abstract:

This paper deals with the modelling of the heat transfer process in a thin porous fibrous material such as a paper sheet when it is subjected to an incident heat flux introduced by a laser beam. A mathematical model based on the control volume principle is developed for numerical estimation of radial temperature distribution which is validated experimentally by infrared thermography. Here the heat flux is introduced by a CO_2 laser beam of 10.6 µm wavelength and an infrared image sequence is recorded as a function of time with a high resolution infrared camera. The preliminary validation results indicate that the simulation model can predict the transient development of sheet temperature very accurately under the specified heating conditions. The model can enhance our understanding and insights of the heat transfer process in such media, which is of great interest for many drying and thermal applications. Though the application shown here is on a 0.1 mm thick paper sheet, the model can be extended to any thin porous fibrous media such as textiles and nonwovens.

Key words:

Heat transfer model, infrared thermography, control volume, convection coefficient, drying

Nomenclature:

A	area perpendicular to the heat flow, m^2	q''_{con}	Convective heat flux, W/m^2
c	speed of light, m/s	q''_{eff}	effective heat flux to paper by laser beam, W/m^2
c_p	isobaric specific heat of paper, J/kg·K	q_{rad}	net radiation heat transfer rate, W
d	paper thickness, m	q''_{rad}	net radiation heat flux, W/m^2
E	emissive power, W/m^2	r	radius of the laser beam, m
E_b	blackbody emissive power, W/m^2	R, ϕ	Cylindrical coordinates, m, rad
$\dot{E}_{eff}$	rate of effective radiation, W	S	surface area, m^2
$\dot{E}_{las}$	rate of radiation generated by the laser, W	t	time, s
$\dot{E}_{ref}$	rate of reflected radiation, W	T	temperature, K
$\dot{E}_{tra}$	rate of transmitted radiation, W	T_p	paper surface temperature, K
h	Planck's constant	T_{sur}	surrounding object surface temperature, K
h_{con}	Convection heat transfer coefficient, W/m^2·K	T_∞	surrounding air temperature far away from the heated surface, K
h_{rad}	radiation heat transfer coefficient, W/m^2·K	x, y	Cartesian coordinates, m
k	Boltzmann's constant	ε	surface emissivity coefficient of an object
k_x, k_y, k_R, k_ϕ	thermal conductivity, W/m·K	λ	wavelength, m
L	distance between two consecutive control volume centres, m	ρ	paper density, kg/m^3
p	number of calculation time steps	σ	Stefan-Boltzmann constant

Introduction

Heat transfer in porous fibrous media is a very complex and classical problem which has been studied extensively. A thorough understanding of this problem is essential for many drying and thermal applications of such materials. One classical application is in the area of the production of paper and nonwoven materials where heat is applied to evaporate excess water. Similarly, many textile materials and nowovens which are used as thermal barrier materials are also subjects for such a study.

Due to the rapid development of infrared imaging techniques, high resolution infrared cameras have found plenty of application to the investigation of dynamic processes such as heat and mass transfer. And Infrared thermography has been used in scientific research as well as many industrial applications. Meola and Carlomagno (2004) did an extensive review onrecent advances in the use of infrared thermography. The application of infrared thermography in chemical engineering has been elucidated by Bolf (2004). In the paper industry, infrared thermography has been employed for detecting paper defects and improving the efficiency of paper machines (Atkins et al. 1982, Mercer 1984). Likewise, thermography has been actively used for the measurement of thermal and conduction properties of many woven textile materials and nonwovens (Michalak et al. 2000, Wiecek et al. 2003).

As thermography is a method of determining the spatial distribution of temperature in objects and its time dependence (Vavilov 1996), it can very well be used to observe the temperature response of materials when subjected to heating or cooling conditions. A straightforward application of infrared thermography is to investigate the heat transfer problem concerning the temperature distribution in plane of the paper. Previously, a measurement technique to characterize the thermal properties of paper based on infrared thermography was reported (Banerjee et al. 2007). In this paper, a mathematical model of the heat transfer process in a thin porous fibrous material such as a paper sheet is developed when it is heated with an external incident heat flux introduced by a CO_2 laser beam. The model output is also compared with the experimentally measured temperature distribution with the help of active thermography. It is worthwhile to mention that though the application shown here is on a 0.1 mm thick paper sheet, the model can be extended to any thin porous fibrous media such as textiles and nonwovens.

Model development

Being a cellulosic hygroscopic material, paper absorbs or desorbs moisture when its temperature or the environmental conditions change. Heat may be generated or dissipated by paper during the sorption or desorption process. In order to keep the experimental conditions as constant as possible, i. e. the paper moisture at equilibrium state, only very small temperature perturbation is introduced to the paper sample. Under such condition, the total mass as well as the thermal properties of paper can be regarded as constant throughout the experiment.

The heat transfer problem under consideration here involves:

- A radiation heat flux from an infrared CO2 laser of 10.6 µm wavelength to the paper surface.
- The irradiant energy is partially absorbed by the paper to increase its temperature, partially reflected by the paper surface to the environment, and partially transmitted through the paper sheet.
- The heat absorbed by the paper may be transferred in paper away from the heated area by conduction; it can also be transferred to the environment by radiation and natural convection.

Laser radiation energy

The laser radiation power is regulated by changing the duty cycle using a pulse width modulation (PWM) control of the laser. The desired heating or cooling cycle is obtained by switching on or off the input power. Due to the paper surface's reflecting and transmitting through, only part of the irradiant energy is absorbed by the paper to increase its temperature. The rate of the effective radiant energy is

$$\dot{E}_{eff} = \dot{E}_{las} - \dot{E}_{ref} - \dot{E}_{tra}. \tag{1}$$

Knowing the diameter of the laser beam, the effective heat flux to the paper can be obtained

$$q''_{eff} = \frac{\dot{E}_{eff}}{\pi \cdot r^2}. \tag{2}$$

The rate of laser energy as well as the rate of the transmitted energy can easily be measured with a hand-held laser power meter. The reflected energy can be minimized if the paper surface is lightly coated with graphite spray.

Radiation heat loss

The spectral distribution of the radiation intensity from a blackbody can be described by the thermal radiation law of Planck (Rohlf 1994)

$$\frac{dE_b}{d\lambda} = \frac{2\pi hc^2}{\lambda^5 \left(e^{hc/\lambda kT} - 1\right)}. \tag{3}$$

By integrating the Planck formula (3) over the entire wavelength spectrum, the total power per area radiated from the blackbody surface can be obtained:

$$E_b = \sigma T^4, \tag{4}$$

where, the Stefan-Boltzmann constant is defined as $\sigma \equiv \frac{2\pi^5 k^4}{15 h^3 c^2} = 5.67 \times 10^{-8}$ W•m-2•K-4. And Equation (4) is known as the Stefan-Boltzmann law. It gives the maximum power radiated by a body at a given temperature which may only be achieved on a blackbody surface. For a real object which is not an ideal absorber, an emissivity coefficient has to be introduced into the Stefan-Boltzmann law

$$E = \varepsilon \sigma T^4. \tag{5}$$

The emissivity is defined to be the fraction of incident radiation energy absorbed by the object. And the emissivity is temperature dependent. This is another reason why a small temperature perturbation is desired during the experiment. Of course, a small temperature perturbation requires a high precision infrared camera. With the available technology, a modern infrared camera can sense a temperature down to several millikelvins. For example, the infrared camera used for the measurements described in this paper is a Raytheon Amber Radiance I MWIR camera. Its InSb detector is sensitive in the wavelength range of 3 - 5 µm. The noise level of the camera is characterized by a Noise Equivalent Temperature Difference (NETD) of 25 mK. The detector is a square Focal Plane Array (FPA) with a resolution of 256 × 256 pixels. The camera is capable of a frame rate of 60 Hz.

As for a piece of paper hanging in the air, the net rate of radiation heat transfer per unit area of the paper surface to the environment is (Incropera, DeWitt 2007)

$$q''_{rad} = \varepsilon\sigma\left(T_p^4 - T_{sur}^4\right). \tag{6}$$

If we define the radiation heat transfer coefficient as

$$h_{rad} \equiv \varepsilon\sigma\left(T_p + T_{sur}\right)\left(T_p^2 + T_{sur}^2\right), \tag{7}$$

the net radiation heat exchange can then be conveniently expressed in this form

$$q_{rad} = h_{rad} S\left(T_p - T_{sur}\right). \tag{8}$$

In the following discussion, the radiative heat loss from the paper sample is accounted for by the radiation heat transfer coefficient, which is temperature dependent according to (7). Moreover, the radiation heat transfer coefficient also depends on the emissivity of paper. It is, however, very convenient to formulate the radiation heat transfer coefficient in this manner. The convenience is rather evident in the following section concerning the numerical solution of the heat transfer problem.

Convection heat loss

In present setup, the paper is placed horizontally on a supporting frame. When the paper is heated up by a laser beam, the heat in the paper may be transferred away by natural convection. According to Newton's law of cooling (Jiji 2006), the convective heat flux is proportional to the temperature difference between the paper surface and the air temperature far away from the surface,

$$q''_{con} = h_{con}\left(T_p - T_\infty\right). \tag{9}$$

The convection heat transfer coefficient is of great interest to researchers and engineers who are solving convective heat transfer problems. In fact, natural convection adjacent to horizontal plates, as is the case of present setup, is a classical heat and mass transfer problem, which has been studied extensively by researchers, e. g. Goldstein et al. (1973) and Lloyd and Moran (1974). They derived correlation equations from their experimental data obtained under various conditions. These empirical equations are very useful in solving practical engineering problems, and they have been cited by textbooks on the subject of heat and mass transfer (Incropera, DeWitt 2007; Jiji 2006; Kreith, Bohn 2001; Cengel 2007). Unfortunately, these correlation equations are valid for the case of horizontal plates with uniform surface temperature. This is not true for our setup, where the heated area is only a very small part of a paper sheet. It will be shown that the simulation model can be used for evaluating the convection heat transfer coefficient.

Heat conduction in paper

Since paper is such a thin planar with thickness of 0.1 mm or less for a common paper grade, the heat propagation in the thickness direction from one surface to the other surface takes place in less than a dozen of milliseconds. Whilst the maximum sampling rate of the infrared camera used in our investigation is 60 Hz, and it is not of our current interest to observe the transient heat transfer in the thickness direction. Therefore, the heat transfer inside paper can be considered as two-dimensional in the paper plane. Assuming no internal heat generation in the paper during the investigation conditions, the heat conduction in the paper can be described by

$$\rho c_p \frac{\partial T_p}{\partial t} = \frac{\partial}{\partial x}\left(k_x \frac{\partial T_p}{\partial x}\right) + \frac{\partial}{\partial y}\left(k_y \frac{\partial T_p}{\partial y}\right). \tag{10}$$

Because a round laser beam is used to heat up the paper specimen in our setup, it is more convenient to express the heat conduction in the paper plane in cylindrical coordinates

$$\rho c_p \frac{\partial T_p}{\partial t} = \frac{1}{R}\frac{\partial}{\partial R}\left(k_R R \frac{\partial T_p}{\partial R}\right) + \frac{1}{R^2}\frac{\partial}{\partial \phi}\left(k_\phi \frac{\partial T_p}{\partial \phi}\right). \tag{11}$$

As far as a handsheet is concerned, the paper can be considered as homogeneous because the fibres are randomly distributed in the paper plane. Consequently, the circumferential heat transfer can be neglected. The paper plane can then be divided into radial sectors centred at the axis of the incident laser beam. And the in-plane heat conduction equation, Equation (11), reduces to

$$\rho c_p \frac{\partial T_p}{\partial t} = \frac{1}{R}\frac{\partial}{\partial R}\left(k_R R \frac{\partial T_p}{\partial R}\right). \tag{12}$$

If a heterogeneous paper sheet, e.g. machine-made sheet or some nonwoven material, is to be investigated, the two-dimensional conduction equation (11) may be employed instead of (12). In the following discussion, we will focus on one-dimensional heat conduction in paper. However, the method to be developed can be easily generalized for the case of multi-dimensional heat transfer.

Numerical solution method

It is rather difficult to analytically solve the heat transfer problem proposed here which involves all the possible fundamental heat transfer mechanisms, i.e. conduction, convection and radiation. Furthermore, all these modes of heat transfer intertwine and form a complex system. A numerical method seems to be a promising alternative to solve the heat transfer problem in such a system.

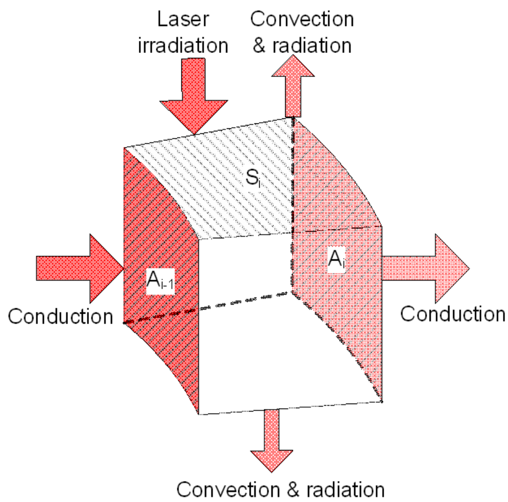

Figure 1. A control volume in paper plane for heat transfer analysis.

We may consider a control volume in paper plane as shown in Figure 1. The control volume is obtained by dividing the paper plane into sections (circular sectors) bounded by concentric circles centred at the laser beam spot. The laser irradiation takes place within the laser illuminated area. Applying energy conservation law (the first law of thermodynamics) to the control-volume, one obtains

$$\frac{\rho c_p S_i d \Delta T_i}{\Delta t} = q_{eff} + q_l - q_r - q_{rad} - q_{con}, \qquad (13)$$

where ΔT_i is the temperature change of the ith control volume in a time interval Δt, i.e. $\Delta T_i = T_i^{p+1} - T_i^{p}$. According to Fourier's law, the conduction heat transfer rates q_l and q_r can be expressed as

$$q_l = k_R A_{i-1} \frac{T_{i-1} - T_i}{L_{i-}} \qquad (14)$$

and

$$q_r = k_R A_i \frac{T_i - T_{i+1}}{L_{i+}}. \qquad (15)$$

If a forward discretization scheme is adopted and the paper plane is assumed as semi-infinite starting from the centre of the laser spot, a system of linear equations describing the heat transfer problem can be derived and formulated in matrix format

$$\mathbf{B} \cdot \mathbf{T} = \mathbf{C}, \qquad (16)$$

where,

$$\mathbf{B} = \begin{bmatrix} F(1) & -F_3(1) & 0 & \cdots & 0 \\ -F_1(2) & F(2) & -F_3(2) & \ddots & \vdots \\ 0 & \ddots & \ddots & \ddots & 0 \\ \vdots & \ddots & -F_1(n-1) & F(n-1) & -F_3(n-1) \\ 0 & \cdots & 0 & -F_1(n) & F(n) \end{bmatrix}, \qquad (17)$$

$$\mathbf{T} = \left[T_1^{p+1} \; T_2^{p+1} \; \cdots \; T_{n-1}^{p+1} \; T_n^{p+1} \right]', \qquad (18)$$

and,

$$\mathbf{C} = \begin{bmatrix} F_2(1) T_1^p \\ F_2(2) T_2^p \\ \vdots \\ F_2(n-1) T_{n-1}^p \\ F_2(n) T_n^p \end{bmatrix} + \left(h_{rad} T_{sur} + h_{con} T_\infty \right) \begin{bmatrix} S_1 \\ S_2 \\ \vdots \\ S_{n-1} \\ S_n \end{bmatrix} + q_{eff} \cdot B_{c1}(p \cdot \Delta t) \cdot B_{c2}(R). \qquad (19)$$

In Equations (16)-(19), the symbol *p* indicates the number of calculation time steps. The absolute time at any given calculation moment can be obtained by multiplying p with the discritization time interval Δt. F, F1, F2 and F3 are constants related to the material physical properties as well as the discritization scheme used. Specifically, $F = \sum_{j=1}^{3} F_j$,

$$F_1(i) = \begin{cases} 0, & i = 1 \\ \dfrac{k_R A_{i-1}}{L_{i-}}, & 1 < i \le n \end{cases}, \quad F_2(i) = \frac{\rho c_p S_i d}{\Delta t}, \quad i \ni [1, n],$$

and $F_3(i) = \dfrac{k_R A_i}{L_{i+}}$. B_{c1} and B_{c2} are boxcar functions, or rectangle functions. They together define when and which control volume surface is heated by the laser beam.

To solve this heat transfer problem described by the system of linear equations (16), an initial condition is required, i.e. the initial temperature distribution in the paper plane T_i^0. This can be precisely measured by an infrared camera. For this purpose, the infrared camera needs to be calibrated. The calibration procedure of the camera is described elsewhere (Popp 2006).

Experimental Validation

The detailed experimental setup and conditions for the validation of the model were described by Banerjee et al. (2007). It is sufficient here to understand that the paper sheet is placed horizontally on a metal frame. A CO2 laser beam of 10.6 micrometer wavelength is vertically projected on the paper sheet. The laser beam diameter and its profile has been measured and considered in the calculation. A calibrated infrared camera captures the images from the same side as the laser and is in perfect synchronization with the laser, i.e., triggered by the laser. The numerical model developed in previous section for the given situation is implemented in MATLAB code for simulation studies. The two-dimensional temperature response on the paper surface during the laser heating and natural cooling process has been captured by a high resolution infrared camera. The temperature data are used for the validation of the simulation model.

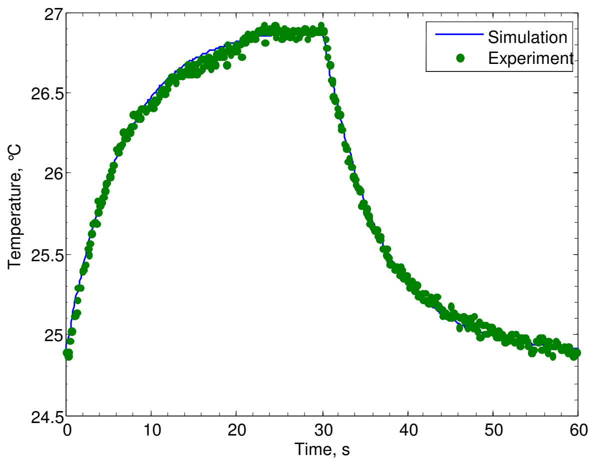

Figure 2. Temperature response of paper subjected to laser heating and natural cooling.

Measurements were carried out for different laser switching frequencies. The simulation result is compared to the measurement data as shown in Figure 2. The measurement temperature data were obtained by evenly heating a paper sheet with a laser source for 30 seconds, then cooling it for another 30 seconds. During the simulation, all parameters can be either measured or calculated, except the natural heat convection coefficient h_{con} which must be estimated by the simulation program. By trial and error, it was found that the value 6.5 W/m2•K gave the best fit between the measurement and the calculation. The correlation coefficient is 0.9987 under such conditions. Such a heat convection coefficient accounts for natural convection from both sides of the paper sheet. Because of the horizontal placement of the paper sheet, the convection from the upper surface is more prominent than that from the bottom surface. According to Jiji (2006), the approximate range of free convection coefficient in gases is 5 - 30 W/m2•K. The present value is very close to the lower range because a small temperature perturbation, only two degrees, was introduced to the paper during the experiment.

Conclusion

In this paper, a numerical model of heat transfer has been developed for the case of a thin fibrous material, such as paper, subjected to heating by an external radiation heat source, e.g. a CO_2 laser. The model has been implemented in MATLAB code. The simulation model has been preliminarily validated with temperature data recorded by an infrared camera. The validation results indicate that the model can give very good estimation of the system under consideration. Additionally, the model can also give an estimation of the free heat convection coefficient, which is usually neither readily available in the literature nor easily measurable by experiment. In this paper, only a preliminary validation is made to demonstrate the validity and capability of the model developed. A thorough validation as well as deeper exploration of model potentials, e. g. in estimation of the thermal properties of porous fibrous material with spatial resolution, is part of future research in this direction.

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